

## Problem 2.43

[Difficulty: 2]

**2.43** Crude oil, with specific gravity  $SG = 0.85$  and viscosity  $\mu = 2.15 \times 10^{-3} \text{ lbf} \cdot \text{s}/\text{ft}^2$ , flows steadily down a surface inclined  $\theta = 45$  degrees below the horizontal in a film of thickness  $h = 0.1 \text{ in}$ . The velocity profile is given by

$$u = \frac{\rho g}{\mu} \left( hy - \frac{y^2}{2} \right) \sin \theta$$

(Coordinate  $x$  is along the surface and  $y$  is normal to the surface.) Plot the velocity profile. Determine the magnitude and direction of the shear stress that acts on the surface.

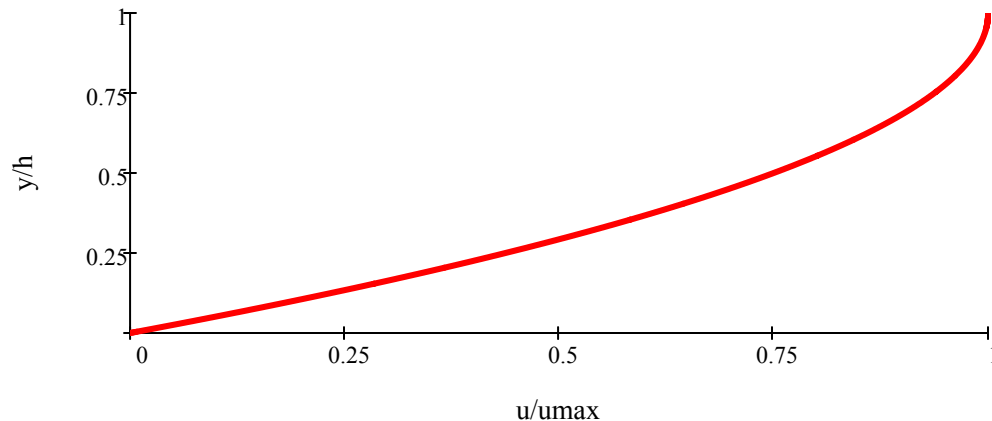
**Given:** Velocity profile

**Find:** Plot of velocity profile; shear stress on surface

**Solution:**

The velocity profile is  $u = \frac{\rho \cdot g}{\mu} \cdot \left( h \cdot y - \frac{y^2}{2} \right) \cdot \sin(\theta)$  so the maximum velocity is at  $y = h$   $u_{\max} = \frac{\rho \cdot g}{\mu} \cdot \frac{h^2}{2} \cdot \sin(\theta)$

Hence we can plot  $\frac{u}{u_{\max}} = 2 \cdot \left[ \frac{y}{h} - \frac{1}{2} \cdot \left( \frac{y}{h} \right)^2 \right]$



This graph can be plotted in *Excel*

The given data is  $h = 0.1 \cdot \text{in}$   $\mu = 2.15 \times 10^{-3} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}$   $\theta = 45 \cdot \text{deg}$

Basic equation  $\tau_{yx} = \mu \cdot \frac{du}{dy}$   $\tau_{yx} = \mu \cdot \frac{du}{dy} = \mu \cdot \frac{d}{dy} \frac{\rho \cdot g}{\mu} \cdot \left( h \cdot y - \frac{y^2}{2} \right) \cdot \sin(\theta) = \rho \cdot g \cdot (h - y) \cdot \sin(\theta)$

At the surface  $y = 0$   $\tau_{yx} = \rho \cdot g \cdot h \cdot \sin(\theta)$

Hence  $\tau_{yx} = 0.85 \times 1.94 \cdot \frac{\text{slug}}{\text{ft}^3} \times 32.2 \cdot \frac{\text{ft}}{\text{s}^2} \times 0.1 \cdot \text{in} \times \frac{1 \cdot \text{ft}}{12 \cdot \text{in}} \times \sin(45 \cdot \text{deg}) \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}}$   $\tau_{yx} = 0.313 \cdot \frac{\text{lbf}}{\text{ft}^2}$

The surface is a positive  $y$  surface. Since  $\tau_{yx} > 0$ , the shear stress on the surface must act in the plus  $x$  direction.